Quiz 1 **Name:** _____
Date: 02/07/2024
ESE 531: Spring 2024
Instructor: Yousef El-Laham

Instructions:
Please answer each part in the question below. You have 15 minutes.

Question 1. (5 points) The standard error of a device is assumed to be distributed according to a Rayleigh distribution:

$$p(x;b) = \begin{cases} \frac{x}{b}\exp\left(\frac{-x^2}{2b}\right), & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

The population mean is given by :

$$\mathbb{E}[X] = \sqrt{\frac{\pi b}{2}}$$

Consider the random sample $X_1, \ldots, X_n \overset{i.i.d.}{\sim} p(x;\sigma)$.

(a) (2 points) Determine the method of moments estimator of σ. Explain your steps and determine if the estimator is biased.

(b) (2 points) Determine the maximum likelihood estimator of σ. Explain your steps and determine if the estimator is biased.

(c) (1 point) The Rayleigh distribution can alternatively be parameterized by a scale parameter $\sigma = \sqrt{b}$:

$$p(x;\sigma) = \begin{cases} \frac{x}{\sigma^2}\exp\left(\frac{-x^2}{2\sigma^2}\right), & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

with expected value $\mathbb{E}[X] = \sigma\sqrt{\pi/2}$. Under this parameterization, what is the maximum likelihood estimator of σ?

Additional page for work.